ABSTRACT: In the study of dynamical systems, mathematicians attempt to understand the long term behavior of systems which evolve in time by a mathematical set of rules. A polygon in the plane determines such a system via idealized billiard trajectories. The study of billiards in polygons gives rise to beautiful connections between the fields of dynamical systems, geometry, algebra, and number theory. In this talk, I will describe some of the motivating questions in the study of billiards, some of the celebrated results in this area, as well as some of the connections to other areas of mathematics.