The Locker Problem

One hallway of Houston High School contains 20 empty lockers, numbered from 1 to 20. At the end of each day all of the lockers are closed. At their morning meeting the 20 members of the math club decides to run an experiment. The first member of the club opened every locker; the second student started with locker #2 and closed every second locker. The third student then started with locker #3 and changed the state of every third locker (opened the locker if it was closed, closed the locker if it was open).The fourth student started with locker #4 and changed the state of every fourth locker, the fifth student started with locker #5, and so on, until all 20 students had passed by the lockers.

Which lockers are still open after the twentieth student is finished? Which locker or lockers changed the most?

Suppose there are 200 lockers and 200 students. Which lockers are open after the 200^{th} student is finished? Which locker or lockers changed the most?

Adapted from *Fostering Algebraic Thinking* by Mark Driscoll (1999). Published by Heinemann